Antiderivatives (2)

Antiderivative of cos(x)

$$\int \cos x \, dx = \sin x + c$$

$$\sin'(x) = \cos(x)$$

Antiderivatives of cos(ax+b)

$$\int \boldsymbol{cos}(\boldsymbol{ax}+\boldsymbol{b})\,\boldsymbol{dx}$$

$$=\frac{\boldsymbol{1}}{\boldsymbol{a}}\boldsymbol{sin}(\boldsymbol{a}\,\boldsymbol{x}+\boldsymbol{b})+\boldsymbol{c}$$

$$sin'(ax+b)$$
$$=a\cos(ax+b)$$

Examples

$$\int \cos(3x)\,dx = \frac{1}{3}\sin(3x) + c$$

$$\int \cos(5x-1)\,dx = \frac{1}{5}\sin(5x-1) + c$$

$$\int [7\cos(4x) + 5\cos(2x)]\,dx = \frac{7}{4}\sin(4x) + \frac{5}{2}\sin(2x) + c$$

Antiderivatives of sin(ax+b)

$$\int \sin(ax+b)\,dx$$

$$= \frac{-1}{a}\cos(ax+b)+c$$

$$\cos'(ax+b) = -a\sin(ax+b)$$

Examples

$$\int \sin(4x)\,dx = \frac{-1}{4}\cos(4x) + c$$

$$\int \sin(7x-3)\,dx = \frac{-1}{7}\cos(7x-3) + c$$

$$\int [7\sin(4x) + 5\sin(2x)]\,dx = \frac{-7}{4}\cos(4x) + \frac{-5}{2}\cos(2x) + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int (1 + \boldsymbol{tan^2\, x}) \boldsymbol{dx} = \boldsymbol{tan\, x + c}$$

Example

$$\int \boldsymbol{tan^2 x\, dx} = \int (\boldsymbol{1 + tan^2 x - 1}) \boldsymbol{dx} = \int (\boldsymbol{1 + tan^2 x}) \boldsymbol{dx} - \int \boldsymbol{1.dx}$$

$$\boldsymbol{= tanx - x + c}$$

Linearization formulas

$$\cos(a).\cos(b) = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin(a).\sin(b) = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$

$$\sin(a).\cos(b) = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos(a).\sin(b) = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$$

Application Linearize each of the following functions then find its integral

- $f(x) = \cos(2x)\cos(5x)$
- $g(x) = \sin(3x)\sin(x)$
- $h(x) = \sin(5x)\cos(2x)$
- $k(x) = sin^3 x$
- $t(x) = cos^2(2x)\, cos(3x)$

$$\int U'U^n dx = \frac{U^{n+1}}{n+1} + c$$

- $\int (3x^2 - 2)(x^3 - 2x)^4 dx$
- $\int (5x + 1)(\frac{5}{2}x^2 + x - 1)^7 dx$
- $\int \cos(x) sin^4(x) dx$
- $\int -\sin(x)\, cos^5(x) dx$

Pause the video to solve this application

Solution

- $\int (3x^2 - 2)(x^3 - 2x)^4 dx = \frac{(x^3-2x)^5}{5} + c$
- $\int (5x + 1)\left(\frac{5}{2}x^2 + x - 1\right)^7 dx = \frac{(\frac{5}{2}x^2+x-1)^8}{8} + c$
- $\int \cos(x)\, sin^4(x) dx = \frac{sin^5(x)}{5} + c$
- $\int -\sin(x)\, cos^5(x) dx = \frac{cos^6(x)}{6} + c$